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© World Scientific Publishing Company**USING (4+1) SPLIT AND ENERGY CONDITIONS TO STUDY
THE INDUCED MATTER IN 5D RICCI-FLAT COSMOLOGY**

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We use $(4 + 1)$ split to derive the $4D$ induced energy density ρ and pressure p of the $5D$ Ricci-flat cosmological solutions which are characterized by having a bounce instead of a bang. The solutions contain two arbitrary functions of time t and, therefore, are mathematically rich in giving various cosmological models. By using four known energy conditions (null, weak, strong, and dominant) to pick out and study physically meaningful solutions, we find that the $4D$ part of the $5D$ solutions asymptotically approach to the standard $4D$ FRW models and the expansion of the universe is decelerating for normal induced matter for which all the four energy conditions are satisfied. We also find that quintessence might be normal or abnormal, depending on the parameter w of the equation of state. If $-1 \leq w < -1/3$, the expansion of the universe is accelerating and the quintessence is abnormal because the strong energy condition is violated while other three are satisfied. For phantom, all the four energy conditions are violated. Before the bounce all the four energy conditions are violated, implying that the cosmic matter before the bounce could be explained as a phantom which has a large negative pressure and makes the universe bouncing. In the early times after the bounce, the dominant energy condition is violated while the other three are satisfied, and so the cosmic matter could be explained as a super-luminal acoustic matter.

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1. Introduction

The Campbell-Magaard theorem^{1–2} states that any analytic solution of Einstein's equations in N -dimensions can be locally embedded in a $(N + 1)$ -dimensional Ricci-flat manifold. It was noted^{3–6} that this theorem provides a mathematical support to the 5-dimensional Space-Time-Matter (STM) theory^{7–8} in which the $5D$ manifold is Ricci-flat with $R_{ab} = 0$ while the $4D$ matter is induced from the $5D$ vacuum. Many solutions of this type have been studied within the framework of the STM

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theory. Here, in this paper, we are going to study a class of 5D Ricci-flat cosmological solutions which was firstly presented by Liu and Mashhoon⁹ and restudied latter by Liu and Wesson¹⁰. This class of solutions is algebraically rich because it contains two arbitrary functions of the time t . It was shown¹⁰ that two major properties characterize these 5D models: Firstly, the 4D induced matter could be described by a perfect fluid plus a variable cosmological term, and by properly choosing the two arbitrary functions both the radiation and matter dominated standard FRW models could be recovered. Secondly, the big bang singularity of the 4D standard cosmology could be replaced by a big bounce at which the size of the universe is finite and the universe contracts before the bounce and expands after the bounce. Many further studies could be found in literature such as those about the embeddings of the 5D solutions to brane models^{11–13}, about the accelerating expansion and dark energy of the 5D universe^{14–18}, and about the isometry between the 5D solutions and 5D black holes¹⁹. We aware that it is the two arbitrary functions that makes the 5D solutions mathematically rich in giving various cosmological models and various normal and non-normal induced matters. Therefore, it is useful to look for physical constraints to fix these two arbitrary functions and to study physical properties of various different solutions. This paper is organized as follows: In Section II, we will use the $(4+1)$ split technique to derive the 4D induced matter. In Section III, we will use four known energy conditions (strong, weak, null, and dominant) to study properties of the 4D induced matter. Section IV is a short conclusion.

2. $(4+1)$ split and the induced matter

The 5D Ricci-flat cosmological solutions read

$$dS^2 = B^2 dt^2 - A^2 \left(\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right) - dy^2, \quad (1)$$

$$A^2 = (\mu^2 + k) y^2 + 2\nu y + \frac{\nu^2 + C}{\mu^2 + k}, \quad (2)$$

$$B = \frac{1}{\mu} \frac{\partial A}{\partial t} \equiv \frac{\dot{A}}{\mu}, \quad (3)$$

where $d\Omega^2 = d\theta^2 + \sin^2 \theta d\psi^2$, $\mu = \mu(t)$ and $\nu = \nu(t)$ are two arbitrary functions, k is the 3D curvature index ($k = \pm 1, 0$), C is a constant. Because the 5D manifold (1)-(3) is Ricci-flat, we have $I_1 \equiv R = 0$, $I_2 \equiv R^{ab} R_{ab} = 0$, and

$$I_3 \equiv R^{abcd} R_{abcd} = \frac{72C^2}{A^8}, \quad (4)$$

so C is related to the 5D curvature.

The general $(4+1)$ split in STM theory has been given in Ref. 20, where it was shown that the 15 5D equations $R_{ab} = 0$ decompose into $10 + 4 + 1$ 4D equations: ten for the 4D Ricci tensor, four for the Gauss-Codazzi equations, and one for the

scalar field. For our 5D metric (1) we see that the 4D hypersurfaces Σ_y are $y \equiv$ constant and the normal vectors to Σ_y are

$$n^a = (0, 0, 0, 0, 1), \quad n_a = (0, 0, 0, 0, -1). \quad (5)$$

So the projection tensor is

$$h_{ab} = g_{ab} + n_a n_b. \quad (6)$$

The extrinsic curvature tensor K_{ab} , which describes the rate of changes of Σ_y as it moves in the normal direction, is defined by

$$K_{ab} = -h_a^c \nabla_c n_b, \quad (7)$$

where ∇_c is the 5D covariant derivative operator. Meanwhile, the lapse function $\Phi = 1$ and the shift vector $N^\alpha = 0$. These greatly simplifies the calculations for K_{ab} . Using (5) and (6) we find

$$K_{ab} = \begin{pmatrix} K_{\alpha\beta} & 0 \\ 0 & 0 \end{pmatrix}, \quad (8)$$

with

$$K_{\alpha\beta} = -\frac{1}{2} \partial_y g_{\alpha\beta}. \quad (9)$$

The 4D Ricci tensor ${}^{(4)}R_{\alpha\beta}$ on the 4D hypersurface Σ_y can be expressed in terms of $K_{\alpha\beta}$ by

$${}^{(4)}R_{\alpha\beta} = \partial_y K_{\alpha\beta} - K K_{\alpha\beta} + 2K_{\alpha\gamma} K_{\beta}^{\gamma}, \quad (10)$$

where $K = g^{\alpha\beta} K_{\alpha\beta}$. Substituting the exact 5D solutions (1) - (3) into (9) to calculate $K_{\alpha\beta}$ and then using $K_{\alpha\beta}$ in (10), we obtain the non-vanishing components of ${}^{(4)}R_{\alpha\beta}$ being

$$\begin{aligned} {}^{(4)}R_0^0 &= -\frac{3\mu\dot{\mu}}{A\dot{A}}, \\ {}^{(4)}R_1^1 &= {}^{(4)}R_2^2 = {}^{(4)}R_3^3 = -\left[\frac{\mu\dot{\mu}}{A\dot{A}} + \frac{2(\mu^2 + k)}{A^2} \right]. \end{aligned} \quad (11)$$

So the 4D Einstein tensor ${}^{(4)}G_{\beta}^{\alpha} \equiv {}^{(4)}R_{\beta}^{\alpha} - \delta_{\beta}^{\alpha} {}^{(4)}R/2$ become

$$\begin{aligned} {}^{(4)}G_0^0 &= \frac{3(\mu^2 + k)}{A^2}, \\ {}^{(4)}G_1^1 &= {}^{(4)}G_2^2 = {}^{(4)}G_3^3 = \frac{\mu^2 + k}{A^2} + \frac{2\mu\dot{\mu}}{A\dot{A}}. \end{aligned} \quad (12)$$

According to Einstein' theory of general relativity, this ${}^{(4)}G_{\beta}^{\alpha}$ implies an effective or induced 4D energy-momentum tensor ${}^{(4)}T_{\beta}^{\alpha}$. Let ${}^{(4)}G_{\beta}^{\alpha} = {}^{(4)}T_{\beta}^{\alpha}$ and suppose ${}^{(4)}T_{\beta}^{\alpha}$ being described by a perfect fluid with density ρ and pressure p ,

$${}^{(4)}T_{\alpha\beta} = (\rho + p)u_{\alpha}u_{\beta} - pg_{\alpha\beta}, \quad (13)$$

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where u^α is the 4-velocity with $u^\alpha = (u^0, 0, 0, 0)$ and $u^0 u_0 = 1$, then we find

$$\begin{aligned}\rho &= \frac{3(\mu^2 + k)}{A^2}, \\ p &= -\frac{\mu^2 + k}{A^2} - \frac{2\mu\dot{\mu}}{A\dot{A}}.\end{aligned}\quad (14)$$

Thus we rederived the same results for the 4D induced matter as those in Ref. 10. From the 5D solutions (1) - (3) one can see that if the two arbitrary functions $\mu(t)$ and $\nu(t)$ are specified, the two scale factors $A(t, y)$ and $B(t, y)$ and then the evolution of the universe are fixed immediately. Then the cosmic induced matter ρ and p can be calculated with use of (14). Meanwhile, the 4D Bianchi identities ${}^{(4)}G^\alpha_{\beta;\alpha} = 0$ lead to the 4D conservation laws ${}^{(4)}T^\alpha_{\beta;\alpha} = 0$ which give

$$\dot{\rho} = -3\frac{\dot{A}}{A}(\rho + p), \quad (15)$$

as is in the standard 4D cosmology.

3. Energy conditions

The energy conditions of Einstein's general relativity are designed to extract as much information as possible from the field equations. They were used in deriving many theorems such as the singularity theorems²¹, the censorship theorem²² and so on. In this section, we wish to use the energy conditions to study the properties of the induced matter of the 5D Ricci-flat cosmological solutions (1) - (3).

The standard classical energy conditions are the null energy condition (NEC), weak energy condition (WEC), strong energy condition (SEC), and dominant energy condition (DEC). Basic definitions of these energy conditions can be found in Ref. 23. For the case in cosmology they are²⁴⁻²⁵

$$NEC : \quad \rho + p \geq 0, \quad (16)$$

$$WEC : \quad \rho \geq 0 \quad \text{and} \quad \rho + p \geq 0, \quad (17)$$

$$SEC : \quad \rho + 3p \geq 0 \quad \text{and} \quad \rho + p \geq 0, \quad (18)$$

$$DEC : \quad \rho \geq 0 \quad \text{and} \quad \rho \geq |p|. \quad (19)$$

Here, for simplicity, we are not going to discuss models which contain more than one components of matter. We just consider the case where the universe is dominated by just one fluid with ρ and p .

Without loss of generality, we write the equation of state as

$$p = w\rho. \quad (20)$$

Because both ρ and p are functions of t and y in the 5D model, So w is, in general, a function of t and y as well, $w = w(t, y)$. Substituting (20) into the four energy

conditions (16) - (19), we get

$$NEC : \quad (1 + w)\rho \geq 0, \quad (21)$$

$$WEC : \quad \rho \geq 0 \quad \text{and} \quad w \geq -1, \quad (22)$$

$$SEC : \quad (1 + 3w)\rho \geq 0 \quad \text{and} \quad (1 + w)\rho \geq 0, \quad (23)$$

$$DEC : \quad \rho \geq 0 \quad \text{and} \quad |w| \leq 1. \quad (24)$$

Following previous usage (see, for example, Ref. 24-25) we call matter that satisfies all the four energy conditions “normal” and call matter that specifically violates the SEC “abnormal”. And we call matter that violates any one of the four energy conditions “non-normal”. In the following we will discuss both the normal and non-normal matter separately.

3.1. Models dominated by normal matter

For normal matter all the four standard energy conditions should be satisfied. We can easily show that (21) - (24) are equivalent to

$$\text{Normal Matter :} \quad \rho \geq 0 \quad \text{and} \quad -1/3 \leq w \leq 1. \quad (25)$$

With use of these constraints in the 5D induced matter (14) we obtain

$$\mu^2 + k \geq 0, \quad (26)$$

$$-\frac{2\mu\dot{\mu}}{A\dot{A}} \geq 0, \quad (27)$$

$$\frac{2(\mu^2 + k)}{A^2} + \frac{\mu\dot{\mu}}{A\dot{A}} \geq 0. \quad (28)$$

Meanwhile, with use of the definition of the proper time, $d\tau \equiv Bdt$, and the relation $B = \dot{A}/\mu$, the Hubble and deceleration parameters of the 5D solutions are found to be

$$H = \frac{\mu}{A}, \quad q \equiv -\frac{A\dot{\mu}}{\mu\dot{A}}. \quad (29)$$

Using (29) in (26) - (28) we get

$$H^2 + \frac{k}{A^2} \geq 0, \quad (30)$$

$$q \geq 0, \quad (31)$$

$$(2 - q)H^2 + \frac{2k}{A^2} \geq 0. \quad (32)$$

We find that all the results in (25) and (30) - (32) are completely the same as those in 4D standard FRW cosmology for models dominated by normal matter. And we also arrive, from (31), at a conclusion that *the expansion of the universe is decelerating if the universe is dominated by normal matter, and so the gravitational force of normal matter is attractive*. This conclusion valid for both the 4D standard FRW cosmology and the 5D Ricci-flat cosmology.

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As we mentioned above that w in (20) is in general not a constant. Now we assume w being a constant for simplicity. Then Eqs. (14) lead to

$$\frac{(3w+1)\dot{A}}{A} = -\frac{2\mu\dot{\mu}}{\mu^2+k}. \quad (33)$$

Integrating with respect to t , we obtain

$$\mu^2 + k = \frac{C_1}{A^{3w+1}} \quad \text{for } w = \text{constant}. \quad (34)$$

And the conservation law(15) gives

$$\rho = \rho_0 \left(\frac{A_0}{A} \right)^{3(1+w)} \quad \text{for } w = \text{constant}. \quad (35)$$

Let us return to the two arbitrary functions $\mu(t)$ and $\nu(t)$ of the $5D$ solutions. From the $5D$ solution (2) we can see that the second arbitrary function $\nu(t)$ can always be solved out in terms of $A(t)$ if we substituting (34) in (2). This leaves us a freedom to consider the equation (34) alone. Recall that the Campbell-Magaard theorem just says that the $4D$ solutions of Einstein equations can be embedded in a $5D$ Ricci-flat manifold *locally* (not *globally*). This reminds us to choose the time t in such a way that the coordinate time t approaches to the proper time asymptotically for a very large t . Consider the $k = 0$ case for simplicity. Then, to make $B = \dot{A}/\mu \rightarrow 1$, we must have, from (34),

$$\begin{aligned} A &\approx A_0 \left(\frac{t}{t_0} \right)^n, & \mu &\approx \frac{n}{t_0} A_0 \left(\frac{t}{t_0} \right)^{n-1}, \\ n &= \frac{2}{3(w+1)}, & \text{for } t &\gg 0. \end{aligned} \quad (36)$$

Then we get an approximate $5D$ metric

$$\begin{aligned} dS^2 &\approx dt^2 - A_0^2 \left(\frac{t}{t_0} \right)^{2n} (dr^2 + r^2 d\Omega^2) - dy^2, \\ n &= \frac{2}{3(w+1)} \quad \text{for } t \gg 0. \end{aligned} \quad (37)$$

This is a class of approximate solutions of the $5D$ Ricci-flat universe while the corresponding exact solutions are given in (34) and (35). Now we calculate the deceleration parameter q . We find that for both the exact (with $k = 0$) and approximate solutions, we get the same result,

$$q = \frac{3w+1}{2}. \quad (38)$$

As we mentioned above that the constant w takes values from $-1/3$ to 1 for normal matter for which q takes the values from 0 to 2 . So the expansion of the universe is decelerating for normal matter. We should also point out that this approximate solution is actually valid for any values of w as we will discuss in the next subsection. Thus we see from (37) that the $4D$ part of the $5D$ universe asymptotically approaches to that of the standard FRW cosmology in the case for a normal induced matter.

3.2. Models dominated by non-normal matter

Let us consider the 5D approximate solution (36) - (37) and their corresponding exact solutions in (34) and (35) where w takes values from $-1/3$ to 1 for normal matter. Now we discuss the case for $w < -1/3$, for which at least one of the four energy conditions must be violated and so the induced matter is non-normal.

From the four energy conditions (21) - (24) we can see that if $-1 \leq w < -1/3$, SEC is violated while NEC, WEC, and DEC are satisfied. This kind of matter is usually called abnormal matter. We can also see that if $w < -1$, all the four energy conditions, NEC, WEC, SEC, and DEC, are violated. In what follows we will discuss these two cases separately.

3.2.1. Models with $-1 \leq w < -1/3$

For this case, Eq. (38) gives that $-1 \leq q < 0$. So the expansion of the universe is accelerating. Thus we recover the 5D cosmological scaling solution¹⁷ in which the 4D induced matter is of a quintessence model of scalar field. In the quintessence dark energy model²⁶⁻²⁷, w is in the range $[-1, 1]$. So quintessence might be a normal matter if $-1/3 \leq w < 1$ or abnormal matter if $-1 \leq w < -1/3$. This conclusion valid in both the 4D standard and 5D Ricci-flat cosmological models.

3.2.2. Models with $w < -1$

For this case we have $q < -1$. Note that from (36) we find that n is negative and $A(t) \propto t^n$ is decreasing, contrary to the assumption for an expanding universe. To resolve this problem, we find that, without violating their correctness as an approximate solution, the forms of (36) and (37) can be changed to

$$A \approx A_0 \left(\frac{t_m - t_0}{t_m - t} \right)^n, \quad \mu \approx \frac{n}{t_m - t_0} A_0 \left(\frac{t_m - t_0}{t_m - t} \right)^{n-1},$$

$$n = -\frac{2}{3(w+1)}, \quad \text{for } 0 \ll t < t_m. \quad (39)$$

and

$$dS^2 \approx dt^2 - A_0^2 \left(\frac{t_m - t_0}{t_m - t} \right)^{2n} (dr^2 + r^2 d\Omega^2) - dy^2,$$

$$n = -\frac{2}{3(w+1)} \quad \text{for } 0 \ll t < t_m. \quad (40)$$

Thus we recover the 5D attractor solution¹⁸ in which the 4D induced matter is described by a phantom model of the dark energy. As t tends to t_m , the scale factor $A(t)$ tends to infinity implying that the universe will undergo a big rip and will expand to infinity within a finite time. Here we see that all the four energy conditions are violated for phantom,

3.3. Before and during the bounce

The 5D Ricci-flat cosmological models are characterized by having a big bounce instead of a big bang. Now we wish to know what kind of non-normal matter dominated the universe before and during the bounce. To make the discussion easier, we list an exact solution¹⁴ in the following.

$$\begin{aligned} k &= 0, \quad C = 1, \\ \nu(t) &= t_c/t, \quad \mu(t) = t^{-1/2}, \end{aligned} \quad (41)$$

where t_c is a constant. Substituting this equation into (2), (3) and (14), we obtain

$$A^2 = t \left[1 + \left(\frac{y + t_c}{t} \right)^2 \right], \quad (42)$$

$$B^2 = \frac{1}{4} \left[1 - \left(\frac{y + t_c}{t} \right)^2 \right]^2 \left[1 + \left(\frac{y + t_c}{t} \right)^2 \right]^{-1}, \quad (43)$$

and

$$\rho = \frac{3}{t^2 \left[1 + ((y + t_c)/t)^2 \right]}, \quad (44)$$

$$p = \frac{2}{t^2 \left[1 - ((y + t_c)/t)^2 \right]} - \frac{1}{t^2 \left[1 + ((y + t_c)/t)^2 \right]}. \quad (45)$$

From Eq.(42) we can show that the scalar factor $A(t, y)$ has a minimum point at

$$t = |y + t_c| \equiv t_b. \quad (46)$$

at which we have

$$A|_{t=t_b} = (2t_b)^{1/2}, \quad B|_{t=t_b} = 0, \quad \dot{A}|_{t=t_b} = 0. \quad (47)$$

Here $t = t_b$ is the big bounce singularity.

In this solution, the time t is defined in the range $(0, \infty)$. Generally, t is not the proper time. However, when $t \rightarrow \infty$, we have $B \rightarrow 1/4$ and $A \rightarrow t^{1/2}$. so the coordinate time t tends to the proper time for very large t . The bounce is at $t = t_b$. Before the bounce means for $0 < t < t_b$. When $t \rightarrow 0$, we have $A \rightarrow \infty$ and $B \rightarrow 1/(2t)$. When changed to the proper time τ , we find $t \propto e^{2\tau}$. So $t \rightarrow 0$ corresponds to $\tau \rightarrow -\infty$ and $A \rightarrow \infty$.

From this solution we can show that before the bounce (for $0 < t < t_b$) we have

$$\begin{aligned} \rho &> 0, \quad p < 0, \\ \rho + p &< 0, \quad \rho + 3p < 0. \end{aligned} \quad (48)$$

We find that before the bounce all the four energy conditions, NEC, WEC, SEC, and DEC, are violated. Meanwhile, because $\rho > 0$ and $p < -\rho$, so $w < -1$. So, before the bounce, the non-normal induced matter has a large negative value and generates a repulsive force. Then the bounce could be explained as due to this repulsive force and the induced matter could be explained as a phantom.

We can also shown that for $t_b < t < \sqrt{3}t_b$, we have $\rho > 0$, $p > 0$, $\rho + p > 0$, $\rho + 3p > 0$, and $\rho - |p| < 0$. So NEC, WEC and SEC are satisfied while DEC is violated. During this period, the induced cosmic matter is perhaps a super-luminal acoustic matter.

4. Conclusion

In this paper we have used the $(4 + 1)$ split to derive the $4D$ induced energy-momentum tensor of the $5D$ Ricci-flat cosmological solutions. Then we have used the four energy conditions, NEC, WEC, SEC, and DEC, to discuss the physical properties of the induced matter and the various solutions of the $5D$ models. Firstly, we have shown that if the universe is dominated by normal matter, the expansion of the universe is decelerating and the $4D$ part of the $5D$ Ricci-flat universe approaches asymptotically to the $4D$ standard FRW models in late times of the universe. If the universe is dominated by quintessence, the induced matter could be normal or abnormal, depending on the value of w . If $-1/3 \leq w < 1$, all the four energy conditions are satisfied and the quintessence is a normal matter. If $-1 \leq w < -1/3$, only the SEC is violated and so in this range the quintessence is abnormal and the expansion of the universe is accelerating. For phantom, all the four energy conditions are violated. We also find that before the bounce all the four energy conditions are violated, and so the induced matter could be explained as phantom and the bouncing could be explained as due to the repulsive force of the phantom. In the early time after the bounce, DEC is violated while the other three are satisfied, so the induced cosmic matter in this period could be explained as a super-luminal acoustic matter.

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